

RESEARCH ON THE ANALYSIS AND SYNTHESIS OF A MULTI-CONTOUR MECHANISM WITH MULTIPLE WORKING CONFIGURATIONS

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ABSTRACT

The paper analyzes a plane mechanism that has three independent contours as its component from a positional point of view. In this way, four distinct working configurations of the analyzed mechanism are highlighted, for which the variations on the kinematic cycle of some angular parameters that define these configurations are established. Special attention is paid to the variation on the kinematic cycle of the displacement of one of the working pistons of the mechanism. A method of dimensional synthesis of the mechanism is proposed by imposing a value of the stroke of this working piston, under the conditions of obtaining a minimum of the sum of the lengths of the component bar-type elements. The dimensional synthesis is performed using the Sequential Quadratic Programming (SQP) method, by employing the Optimization package, included in the Maple program. The results obtained in three analyzed cases are presented for one of the working configurations of the mechanism. Special attention was paid to verifying the functioning of the mechanism for the dimensions of the elements obtained in the three analyzed cases. These verifications highlighted the precision and efficiency of the synthesis method of the analyzed mechanism.

Keywords: mechanism, positional analysis, synthesis, optimization, Sequential Quadratic Programming

INTRODUCTION

The optimization of dynamic and kinematic performances of various current mechanical systems often requires the use of multi-contour planar mechanisms with a high degree of structural complexity [1-9]. These mechanisms are widely integrated into industrial applications due to their ability to generate complex trajectories and to transmit large forces with structural rigidity superior to single-contour structures [10-17]. Their performance critically depends on the choice of working configurations and the dimensions of the component elements, which places the stages of analysis and dimensional synthesis at the core of their design process [18-29].

The dimensional synthesis of multi-contour planar mechanisms represents a double challenge, both mathematically and from an engineering point of view. Unlike simple single-contour mechanisms, multi-contour structures involve an interconnected set of nonlinear kinematic equations, where the modification of a single geometric dimension simultaneously influences the global behavior of the entire system.

While the analysis phase verifies the behavior of the mechanism for a given working configuration and operating mode, the dimensional synthesis follows the opposite path, determining in this case the lengths of the mechanism elements to meet various imposed requirements. The efficient correlation of these two design stages is often hampered by the risk of non-assembly of the contours in certain phases of the movement.

The paper analyzes a plane mechanism with three independent contours. Four distinct working configurations of the analyzed mechanism are highlighted. Special attention is paid to the variation during operation of the displacement of one of the working pistons of the mechanism. A synthesis method of the analyzed mechanism is proposed by imposing a value of the stroke of this working piston, under the conditions of obtaining a minimum of the sum of the lengths of its bar-type components. The dimensional synthesis is performed using the Sequential Quadratic Programming (SQP) method. A functional simulator has been developed by the authors using Maple [30], for synthesis being used the function NLPsolve included in the Optimization package. The simulations have been realized by imposing three values for the stroke of the working piston, by considering one of the working configurations. Special attention was paid to verifying the precision and efficiency of the proposed synthesis method.

The scope of this paper is to propose an analysis and synthesis method that can be applied regardless of the configuration of the analyzed mechanism and its complexity, ensuring a high degree of precision. The gap addressed by the paper concerns the possibility of using the SQP optimization method in situations where the mechanism under analysis might fail to assemble. The novelty brought by the study proposed in the paper is given by the combination of the symbolic and numerical approach both in the analysis and in the dimensional synthesis of the analyzed mechanism by using the specific functions offered by the Maple program.

MATERIALS AND METHODS

The mechanism under analysis is represented in Figure 1. From its associated graph (Figure 2), it results that the mechanism comprises three independent loops: 0-1-2-3-0, 0-1-2-4-6-0 and 0-1-2-4-5-7-0.

The vector closure equations corresponding to the three independent loops are as follows: $\vec{OA} + \vec{AD} + \vec{DO} = 0$ for the independent contour 0-1-2-3-0, $\vec{OA} + \vec{AB} + \vec{BF} + \vec{FG} + \vec{GO} = 0$ for the independent contour 0-1-2-4-6-0 and $\vec{OA} + \vec{AB} + \vec{BE} + \vec{EI} + \vec{IO} = 0$ for the independent contour 0-1-2-4-5-7-0, respectively.

The following notations are introduced: $OA = l_1$, $AD = l_2$, $BF = l_4$, $EI = l_5$ and $FG = l_6$.

Projecting the three vector equations onto the (Ox) and (Oy) axes yields the following systems of equations:

$$\begin{cases} l_1 \cdot \cos \varphi_1 + l_2 \cdot \cos \varphi_2 - s_3 = 0 \\ l_1 \cdot \sin \varphi_1 + l_2 \cdot \sin \varphi_2 = 0 \end{cases} \quad (1)$$

$$\begin{cases} l_1 \cdot \cos \varphi_1 + AB \cdot \cos \varphi_2 + l_4 \cdot \cos \varphi_4 + l_6 \cdot \cos \varphi_6 - b = 0 \\ l_1 \cdot \sin \varphi_1 + AB \cdot \sin \varphi_2 + l_4 \cdot \sin \varphi_4 + l_6 \cdot \sin \varphi_6 - a = 0 \end{cases} \quad (2)$$

$$\begin{cases} l_1 \cdot \cos \varphi_1 + AB \cdot \cos \varphi_2 + BE \cdot \cos \varphi_4 + l_5 \cdot \cos \varphi_5 + s_7 - b = 0 \\ l_1 \cdot \sin \varphi_1 + AB \cdot \sin \varphi_2 + BE \cdot \sin \varphi_4 + l_5 \cdot \sin \varphi_5 - a = 0 \end{cases} \quad (3)$$

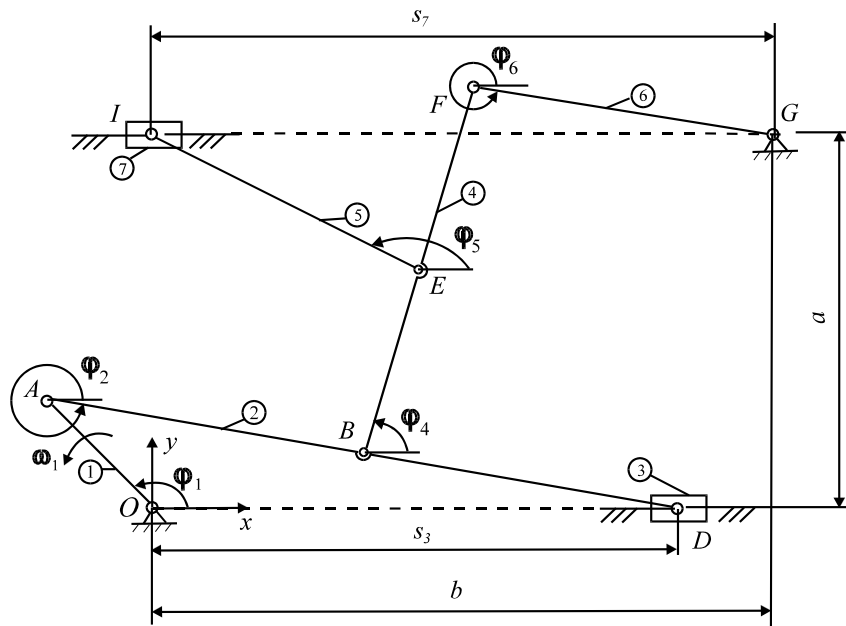


Figure 1. Multi-contour plane mechanism.

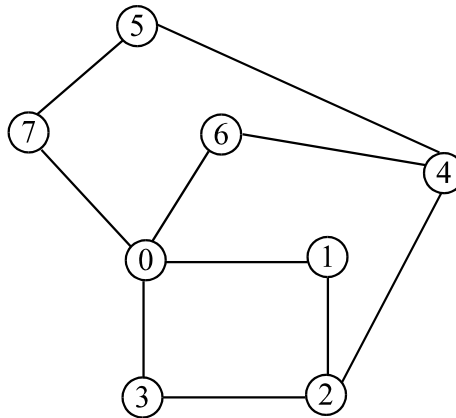


Figure 2. The associated graph.

From the system (1), it results the angle φ_2 and the displacement s_3 :

$$\begin{cases} \varphi_2 = \sin^{-1} \left(-\frac{l_1}{l_2} \cdot \sin \varphi_1 \right) \\ s_3 = l_1 \cdot \cos \varphi_1 + \sqrt{l_1^2 \cdot \cos^2 \varphi_1 + l_2^2 - l_1^2} \end{cases} \quad (4)$$

Solving the system (2), it results the angles φ_4 and φ_6 :

$$\begin{cases} \varphi_4 = (-1)^k \cdot \sin^{-1} \left(C_4 / \sqrt{A_4^2 + B_4^2} \right) + k\pi - \text{ATAN2}(A_4, B_4); \quad k \in \mathbb{Z} \\ \varphi_6 = \text{ATAN2}(-l_1 \cdot \sin \varphi_1 - AB \cdot \sin \varphi_2 - l_4 \cdot \sin \varphi_4 + a, -l_1 \cdot \cos \varphi_1 - AB \cdot \cos \varphi_2 - l_4 \cdot \cos \varphi_4 + b) \end{cases} \quad (5)$$

where:

$$\begin{cases} A_4 = 2 \cdot l_1 \cdot l_4 \cdot \cos \varphi_1 + 2 \cdot AB \cdot l_4 \cdot \cos \varphi_2 - 2 \cdot b \cdot l_4 \\ B_4 = 2 \cdot l_1 \cdot l_4 \cdot \sin \varphi_1 + 2 \cdot AB \cdot l_4 \cdot \sin \varphi_2 - 2 \cdot a \cdot l_4 \\ C_4 = l_6^2 - l_1^2 - AB^2 - l_4^2 - a^2 - b^2 - 2 \cdot l_1 \cdot AB \cdot \cos \varphi_1 \cdot \cos \varphi_2 + \\ + 2 \cdot l_1 \cdot b \cdot \cos \varphi_1 + 2 \cdot AB \cdot b \cdot \cos \varphi_2 - 2 \cdot l_1 \cdot AB \cdot \sin \varphi_1 \cdot \sin \varphi_2 + \\ + 2 \cdot l_1 \cdot a \cdot \sin \varphi_1 + 2 \cdot AB \cdot a \cdot \sin \varphi_2 \end{cases} \quad (6)$$

and $ATAN2(y,x)$ compute $\tan^{-1} \left(\frac{y}{x} \right)$ by considering the signs of the its arguments.

Solving the system (3) yields:

$$\begin{cases} \varphi_5 = (-1)^k \cdot \sin^{-1} \left(\frac{-l_1 \cdot \sin \varphi_1 - AB \cdot \sin \varphi_2 - BE \cdot \sin \varphi_4 + a}{l_5} \right) + k\pi; \quad k \in Z \\ s_7 = -l_1 \cdot \cos \varphi_1 - AB \cdot \cos \varphi_2 - BE \cdot \cos \varphi_4 - l_5 \cdot \cos \varphi_5 + b \end{cases} \quad (7)$$

Relations (5) and (7) highlight the fact that there are four operating configurations of the mechanism under analysis.

RESULTS AND DISCUSSIONS

Based on the analysis of the mechanism, the authors developed a simulator of its operation using the Maple program. The dimensions adopted for the bar-type elements are as follows: $l_1 = 0.14 \text{ m}$; $l_2 = 0.74 \text{ m}$; $l_4 = 0.48 \text{ m}$; $l_5 = 0.55 \text{ m}$ and $l_6 = 0.35 \text{ m}$. It was also considered: $a = 0.46 \text{ m}$ and $b = 0.85 \text{ m}$.

Figures 3÷6 show the variations of the angles φ_4 , φ_5 , and φ_6 over the kinematic operating cycle for the four working configurations of the mechanism under analysis.

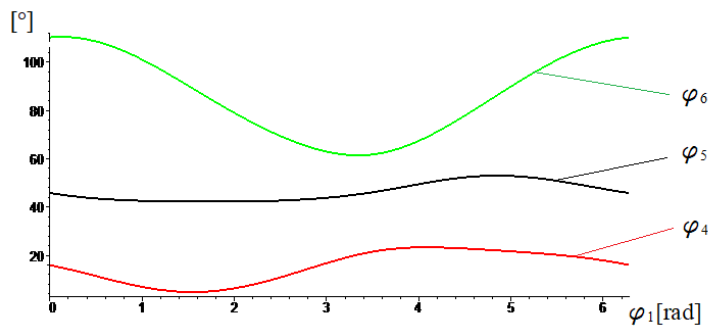


Figure 3. Variation of angles φ_4 , φ_5 , and φ_6 over the kinematic cycle for the mechanism's first operating configuration

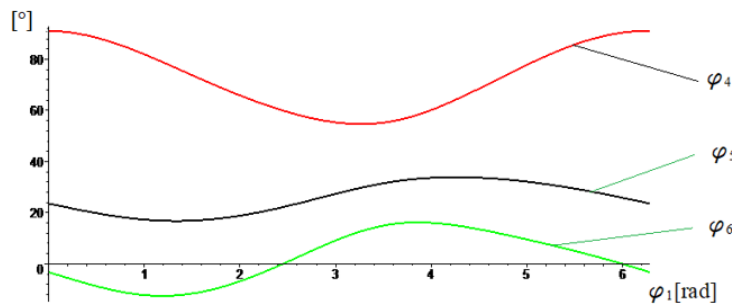


Figure 4. Variation of angles φ_4 , φ_5 , and φ_6 over the kinematic cycle for the mechanism's second operating configuration

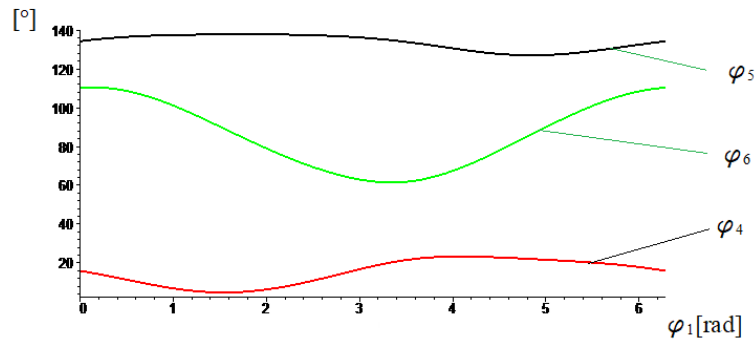


Figure 5. Variation of angles φ_4 , φ_5 , and φ_6 over the kinematic cycle for the mechanism's third operating configuration

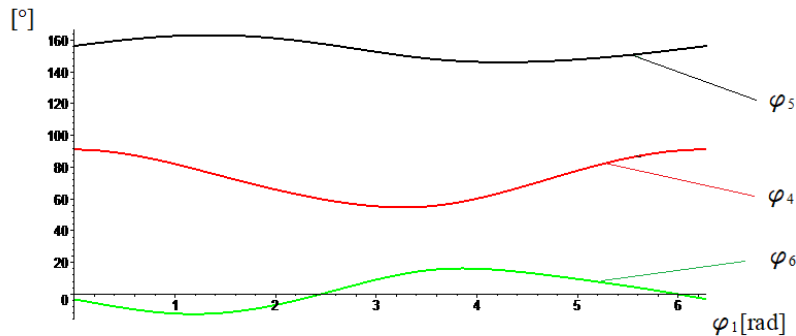


Figure 6. Variation of angles φ_4 , φ_5 , and φ_6 over the kinematic cycle for the mechanism's fourth operating configuration

Special attention was paid to the variation on the kinematic cycle of the displacement of the working piston 7 (Figure 1). A method of dimensional synthesis of the mechanism was proposed by imposing the stroke of this working piston ($s_{7max} - s_{7min}$), under the conditions of obtaining a minimum sum of the lengths of the elements: 1, 2, 4, 5 and 6.

The dimensional synthesis was performed with SQP method, by employing the NLPsolve function, part of the Optimization package from Maple [30]. To call the NLPsolve function, the authors developed a procedure that verifies the mechanism's correct operation by discretizing the working cycle into 360 equidistant points. This enabled the exclusion from the synthesis process of solutions that would have made it impossible to assemble the mechanism in certain working positions.

The simulations have been realized by imposing three values for the stroke of the working piston 7: 0.25 m, 0.3 m and 0.35 m, by considering the third working configuration (Figure 5). In the first case, where the stroke of piston 7 was set to 0.25 m, the following values were obtained for the lengths of the bar-type elements and the dimensions a and b : $l_1 = 0.146126$ m; $l_2 = 0.25$ m; $l_4 = 0.264457$ m; $l_5 = 0.345022$ m, $l_6 = 0.236490$ m, $a = 0.335340$ m and $b = 0.3$ m. The variation of the displacement of piston 7 in this case is shown in Figure 7.

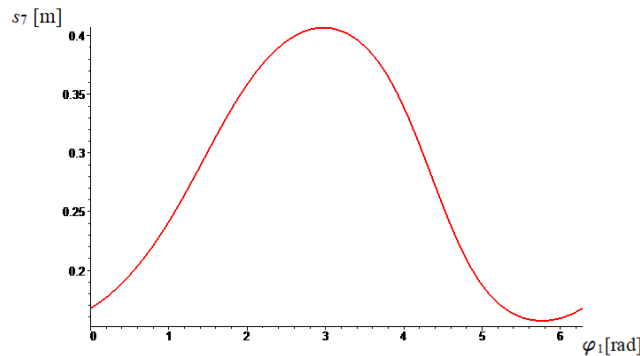


Figure 7. Variation of the displacement of piston 7 for the case where its stroke was set to 0.25 m.

When the stroke of piston 7 was set to 0.30 m, the following values were obtained: $l_1 = 0.143098$ m; $l_2 = 0.320427$ m; $l_4 = 0.283058$ m; $l_5 = 0.296135$ m, $l_6 = 0.210427$ m, $a = 0.241701$ m and $b = 0.435412$ m. The variation of the displacement of piston 7 in this case is shown in Figure 8.

For the third case, when the stroke of piston 7 was set to 0.35 m, the following values were obtained: $l_1 = 0.154300$ m; $l_2 = 0.510736$ m; $l_4 = 0.377039$ m; $l_5 = 0.457529$ m, $l_6 = 0.286741$ m, $a = 0.372782$ m and $b = 0.638613$ m. The variation of the displacement of piston 7 in this case is shown in Figure 9.

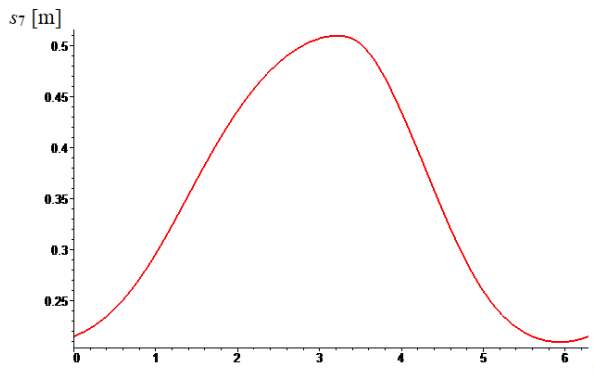


Figure 8. Variation of the displacement of piston 7 for the case where its stroke was set to 0.3 m.

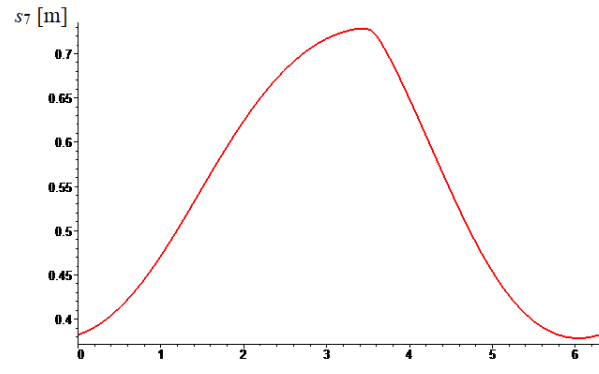


Figure 9. Variation of the displacement of piston 7 for the case where its stroke was set to 0.35 m.

Figures 7, 8, and 9 highlight the precision of the results obtained from the dimensional synthesis of the analyzed mechanism. Specifically, the exact results obtained for the stroke of piston 7 are as follows: 0.2500693586 m, 0.2999996607 m, and 0.3499334295 m, respectively.

CONCLUSIONS

This paper analyzes a multi-loop mechanism that offers four possible operating configurations. Particular attention was paid to the displacement variation of one of the mechanism's working pistons throughout the kinematic cycle, with the aim of achieving working strokes of specified

values. A dimensional synthesis method was developed to achieve the required piston stroke values while minimizing the total length of the bar-type components. The authors implemented the dimensional synthesis, performed using the SQP method, into a functional simulator utilizing the NLPsolve function included in the Optimization package of Maple software. Simulation results, based on one of the mechanism's operating configurations, demonstrated the high precision of the dimensional synthesis method employed. In addition to the method's high precision, its application enables the identification of cases where the mechanism cannot be assembled in certain operating positions. This aspect is crucial for identifying and subsequently eliminating spurious solutions that may arise during the dimensional synthesis process of the mechanism under study. Furthermore, this study methodology is clearly applicable to any of the mechanism's operating configurations and can be further developed to account for other parameters characterizing its operation.

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