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Axial Reactions for a Double Embedded Beam Loaded with an Uniform Pressure

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Abstract

In the paper is presented a way of calculation of the axial reactions for a double embaded beam loaded transversally with an uniform pressure. In order to find the value of the reaction force the differential equation is obtained on the deformed shape of the beam. The final equation is solved nummerically, usisng a specialized programme and the results obtained are exemplified in a calculus example.

Key words: *axial reactions, loads, deformed shape.*

The Principle of the Method

It is considered a double embedded beam with the length l and being loaded by an uniform pressure q (fig. 1) between a and b abscissa.

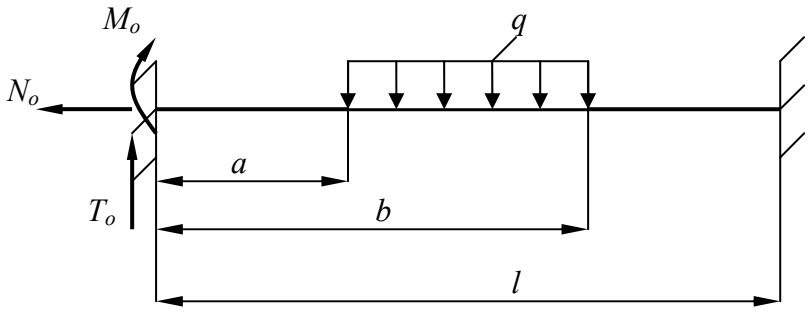

Fig. 1. A double embedded beam loaded with an uniform pressure

Because after the bending the length of the beam rises some horizontal reaction forces are expected to appear at the ends (N_o).

The second order differential equation that describes the bending of the beam is [3]:

$$\frac{d^2v}{dx^2} = -\frac{M(x)}{EI} \quad , \tag{1}$$

where v is the deflection of the current section , $M(x)$ is the bending moment in the same section and EI is the bending rigidity of the beam.

The bending moment from the current section can be written on the deformed shape of the beam under the form :

$$M(x) = M_o + T_o \cdot x - N_o \cdot v(x) \tag{2}$$

Replacing (2) in (1) the following differential equation is obtained:

$$\frac{d^2v}{dx^2} - \frac{N_o}{EI} \cdot v = -\frac{1}{EI}\left(M_o + T_o \cdot x\right) \tag{3}$$

Generally the above differential equation has to verify the following limit conditions (in the origin):

$$x = 0 \Rightarrow \begin{cases} v = v_o \\ \varphi = \varphi_o \end{cases} \text{ a) } ; \quad x = l \Rightarrow \begin{cases} v = v_o \\ \varphi = \varphi_o \end{cases} \text{ b)} \tag{4}$$

In the (4) limit conditions v and φ represent the deflection and the slope of the current section of the beam. For the load case presented in figure (1) is obvious that $v_o = 0$ and $\varphi_o = 0$.

The general solution of the (3) differential equation can be written under the form:

$$v(x) = v_o + \frac{\varphi_o}{2k}\left(e^{kx} - e^{-kx}\right) - \frac{M_o}{2N_o}\left(e^{kx} + e^{-kx} - 2\right) - \frac{T_o}{2kN_o}\left(e^{kx} - e^{-kx} - 2kx\right) + \bar{v} \tag{5}$$

where $\bar{v}$ represents the particular solution that is function of the external loads of the beam.

In order to find the particular solution for the load presented in figure 1 it is considered an uniform pressure that acts on the beam between a and b abscissa. A supplementary current abscissa x_o is considered (fig.2).

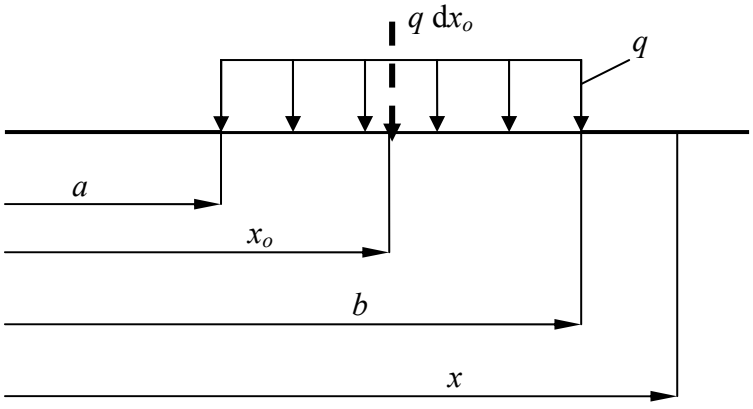

Fig.2. Uniform pressure acting a beam

The suplimentary abscisa x_o allows the calculation of the particular solution $\bar{v}$, by considering a particular term produced by the concentrated force $q\,dx_o$ and integrating over the active area:

- between a and x if $x < b$;
- between a and b if $x > b$.

The particular solution produced by the uniform pressure has to be establishrd by a translation of the starting point :

$$d\bar{v} = dv^q = \frac{q \cdot dx_o}{2 \cdot k \cdot N_o}\left[e^{k(x-x_o)} - e^{-k(x-x_o)} - 2k(x-x_o)\right] \tag{6}$$

Integrating the above relation in respect with x_o over the active area it results:

$$\bar{v} = v^q = \begin{cases} \int_a^x \frac{q}{2 \cdot k \cdot N_o}\left[e^{k(x-x_o)} - e^{-k(x-x_o)} - 2k(x-x_o)\right] \cdot dx_o \\ \int_a^b \frac{q}{2 \cdot k \cdot N_o}\left[e^{k(x-x_o)} - e^{-k(x-x_o)} - 2k(x-x_o)\right] \cdot dx_o \end{cases} = \tag{7}$$

$$\begin{cases} \frac{q}{2kN_o}\left[\frac{1}{k}e^{k(x-a)} + \frac{1}{k}e^{-k(x-a)} - k(x-a)^2 - \frac{2}{k}\right], & x \in [a,b] \\ \frac{q}{2kN_o}\left[\frac{1}{k}e^{k(x-a)} + \frac{1}{k}e^{-k(x-a)} - k(x-a)^2\right] - \frac{q}{2kN_o}\left[\frac{1}{k}e^{k(x-b)} + \frac{1}{k}e^{-k(x-b)} - k(x-b)^2\right], & x > b \end{cases}$$

For the particular case presented in figure 1 the particular solution (7) can be written under the form:

$$\bar{v} = v^q = \frac{q}{2kN_o}\left(\frac{1}{k}e^{kx} + \frac{1}{k}e^{-kx} - kx^2 - \frac{2}{k}\right) \quad , \tag{8}$$

because the beginning abscissa of the uniform pressure is $a = 0$.

Taking into consideration the above relation and the limit conditions (4b) the following equations system is obtained:

$$\begin{cases} M_o\left(e^{kl} + e^{-kl} - 2\right) + T_o \frac{e^{kl} - e^{-kl} - 2kl}{k} = \frac{q}{k^2}\left(e^{kl} + e^{-kl} - k^2l^2 - 2\right) \\ M_o \cdot k \cdot \left(e^{kl} - e^{-kl}\right) + T_o \cdot \left(e^{kl} + e^{-kl} - 2\right) = \frac{q}{k} \cdot \left(e^{kl} - e^{-kl} - 2kl\right) \end{cases} \tag{9}$$

The unknown M_o and T_o can be expressed from (9):

$$M_o = \frac{\frac{q}{k^2}\left[8 - k^2l^2\left(e^{kl} + e^{-kl} + 2\right) + 4kl\left(e^{kl} - e^{-kl}\right) - 4\left(e^{kl} + e^{-kl}\right)\right]}{8 + 2kl\left(e^{kl} - e^{-kl}\right) - 4\left(e^{kl} + e^{-kl}\right)} \tag{10}$$

$$T_o = \frac{\frac{q}{k}\left[k^2l^2\left(e^{kl} - e^{-kl}\right) - 2kl\left(e^{kl} + e^{-kl}\right) + 4kl\right]}{8 + 2kl\left(e^{kl} - e^{-kl}\right) - 4\left(e^{kl} + e^{-kl}\right)} \tag{11}$$

The above relations are function of the unknown N_o because the k coefficient depends on the axial reaction force.

In order to determine the reaction force from the starting point it can be noticed that the difference between the new and the old length of the elastic curve can be expressed by the relations:

$$\Delta l = \frac{N_o \cdot l}{EA} = \int_0^l \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx - l = \int_0^l \sqrt{1+\varphi^2}\,dx - l \tag{12}$$

From the (12) equality it can be obtained the value of N_o only by a nummerical solving. In this respect a specialised programme has been used and the unknown N_o has been obtained for a calculus example.

A Numerical Example

A double embedded beam having the length l = 1m is loaded by an uniform pressure q = 1N/mm over the entire length. The cross sectional area of the beam is a rectangle having the dimensions b x h (40 x 10 mm). The beam is made from steel having the longitudinal elasticity modulus E = 210000 N/mm^2. The abscissa of the beginning of the external pressure is a = 0 and the current abscissa x is always smaller than the length of the beam (fig.3).

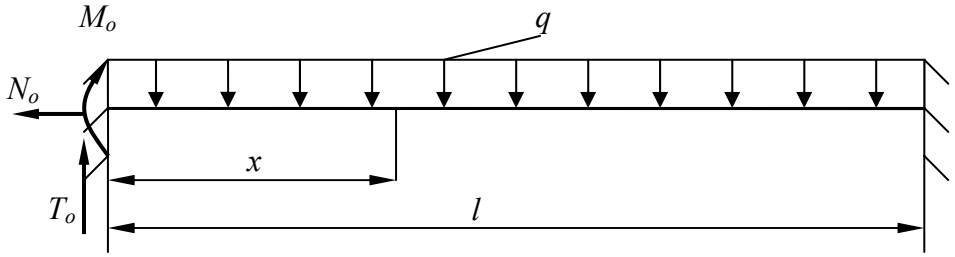

Fig.3. Double embedded beam loaded entirely with uniform pressure

The unknown N_o is the first root of the equation that results from the (12) equality. For the numerical example presented in figure 3 the value of axial reaction force is N_o = 2400 N. With this value it is possible to establish the variations of the deflection, slope, bending moment and shear force along the beam. This variations are presented respectively in figure 4, 5, 6 and 7.

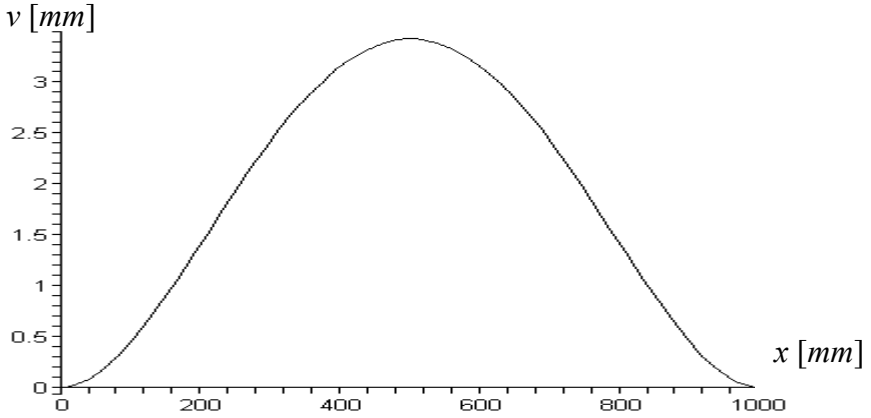

Fig. 4. The deflection curve

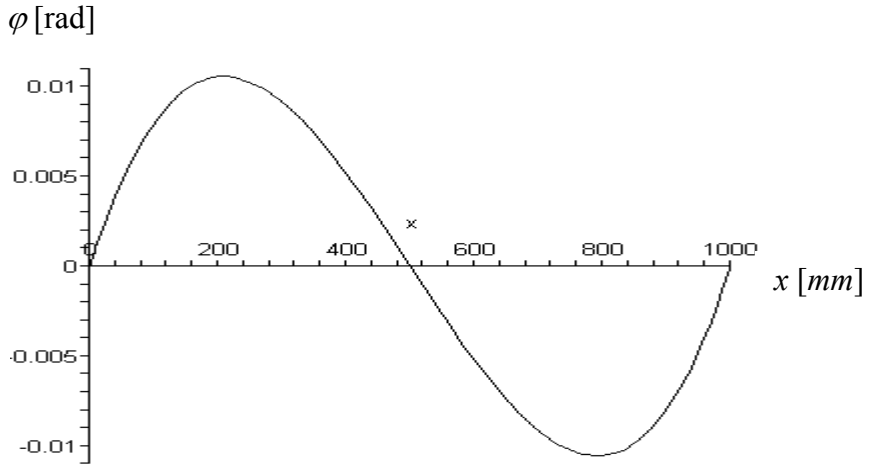

Fig. 5. The variation of the slope

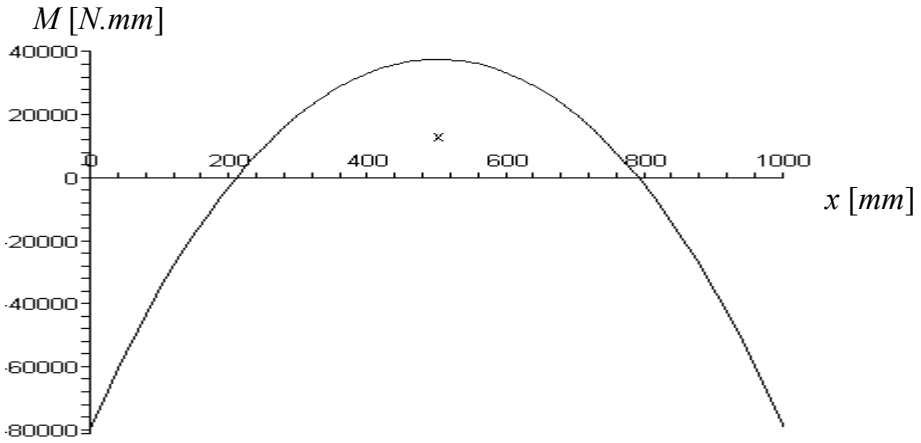

Fig. 6. The variation of the bending moment

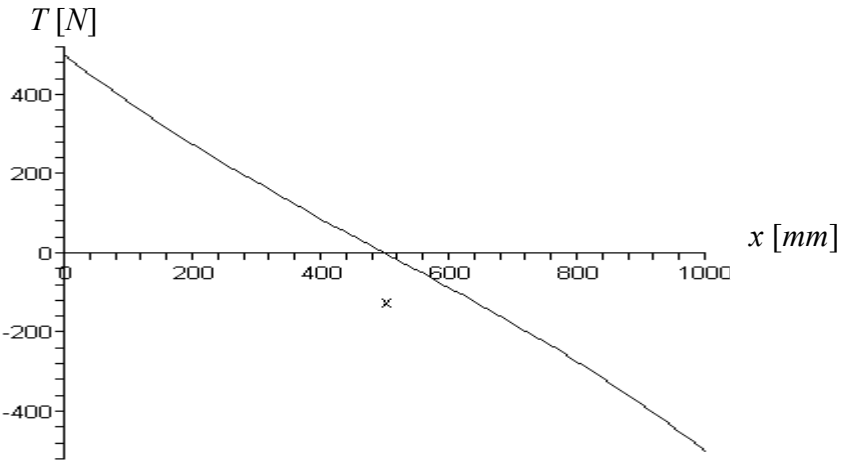

Fig. 7. The variation of the shear force

Analysing the diagrams presented above it can be noticed that the maximum deflection of the beam is 3.427 mm (if the axial reaction force is taken into consideration) and 3.72 mm if the axial reaction force is neglected. Regarding the bending moment, the maximum value is obtained at the ends (M_{max} = 78929.5 N.mm – when the axial reaction force is taken into consideration) and M_{max} = 83333.33 N.mm when the axial reaction force is neglected. The errors obtained are around 8% for the deflection and 5.5% for the bending moment. It is important to notice that the axial reaction force that is obtained produces a composed load that can modify the stress values in every section of the beam. If the rigidity of the beam is lowering (for example by decreasing the depth of the cross sectional area) it is possible that the errors obtained to be more significant.

Conclusions

In the paper it is presented a method of calculation of the axial reaction force that appears when a double embedded beam is transversally loaded with uniform pressure. The results obtained are analysed in a calculus example. The errors between the classical way of calculation (of the efforts and of the displacements of the beam) and the presented method are around 8% but may become higher if the rigidity of the beam decreases.

In order to be more efficient the theoretical results have been transposed in a numerical programme that allows the calculation of the roots of some equations and displays the variations of the efforts and of the displacements.

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Reacțiuni axiale pentru o bară dublu încastrată solicitată cu o presiune uniform distribuită

Rezumat

În lucrare se prezintă o metodă de calcul a reacțiunilor axiale pe o bară dublu încastrată solicitată cu o sarcină uniform distribuită. Rezultatele obținute sunt analizate pe un exemplu de calcul, din care se evidențiază erori de aproximativ 8% (între metoda clasică şi cea prezentată în lucrare). Erorile pot deveni mai importante dacă rigiditatea la încovoiere a barei se micşorează.
Pentru a deveni mai eficientă, metoda prezentată a fost transpusă într-un program de calcul care determină rădăcinile ecuațiilor şi care trasează grafic variațiile eforturilor şi deplasărilor.